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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	November 7, 1994	Final, 9/30/93-9/30/94

4. TITLE AND SUBTITLE
"A Novel Approach to the Numerical Simulation of Turbulent Flows"

5. FUNDING NUMBERS
DAAH04-93-G-0513

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8. PERFORMING ORGANIZATION REPORT NUMBER
003-8620

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
ARO 32412.1-MA-H

11. SUPPLEMENTARY NOTES
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
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12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

During the period covered by this report we have studied the global dynamics of incompressible, viscous fluid motion. This activity included a study of discrete dynamical systems generated by temporal and spatial discretizations of the Navier-Stokes equations. We investigated the existence of absorbing sets and attractors associated with the two dimensional Navier-Stokes equations. The dynamical properties of various finite-difference and finite-element schemes for these equations were also considered. Our goal was to investigate the existence of spurious steady-state solutions.

14. SUBJECT TERMS
Optimal control, nonlinear boundary condition, finite element method, error estimate, von Karman equations

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

Statement of Problems Studied

During the period covered by this report we have studied the global dynamics of incompressible, viscous fluid motion. This activity included a study of discrete dynamical systems generated by temporal and spatial discretizations of the Navier-Stokes equations. We investigated the existence of absorbing sets and attractors associated with the two dimensional Navier-Stokes equations. The dynamical properties of various finite-difference and finite-element schemes for these equations were also considered. Our goal was to investigate the existence of spurious steady-state solutions. A resolution of the issues investigated here could have enormous impact on novel approaches to the numerical simulation of turbulent flows.

A second research project partially supported by this grant included the development of mathematical theories, numerical methods, and supercomputing algorithms for the control of fluid flows. Primary areas of study (with the corresponding application of interest) have been: Magnetohydrodynamic (MHD) flows (fusion plasma); thermally coupled mixed fluid flows (chemical vapor deposition); and free surface flows (polymer extrusion). Results from the study of these problems are important to many Army applications. For example, this work could have an impact on the development of: improved battery components; electromagnetic armor; and advanced electric gun propulsion technology.

Summary Of The Most Important Results

Our initial investigations were concentrated on the control of electrically conducting flows. A simplified model, involving the velocity, pressure and electrical potential fields, was considered. This model is valid in a variety of applications, e.g., in the modelling of a two-dimensional magnetic pump with electrodes on the bounding surface, in the modelling of liquid lithium blanket in fusion technology and metal melt cooling processes. In the isothermal case we have obtained mathematical results such as the existence of optimal solutions and justification of the use of Lagrange multiplier rules to derive an optimality system of equations. We have also developed effective numerical algorithms and successfully implemented these algorithms in the case of simple geometries. See attached computational results shown in Figures MHD a) -- f).

In addition, we have obtained results for control problems for free surface flows in the context of extrusion processes for viscoelastic fluids. The two control objectives investigated were: (1) to obtain a quasi-uniform temperature distribution; and (2) to obtain a desired shape for the free surface. The means of controls were heating/cooling on the bounding surface/extrusion die and adjusting the extrusion speed. Several of these results were reported in the a paper on control in visco-elastic extrusion processes [1]. We also obtained analytical and computational results for the optimal control of the steady-state von Karman equations with distributed controls. Existence of optimal solutions was established. Lagrange multipliers were used to enforce the constraints and to derive an optimality system. Finite element approximations of solutions for the optimality system were developed and corresponding error estimates were derived. See [2] and [3].

Finally, we obtained results on an optimal control problem that is useful for the reduction or prevention of chemical corrosion. The optimal control problem used here was developed for impressed cathodic systems in electrochemisty. The control in this problem was the current density on the anode. A matching objective functional was considered. We first demonstrated the existence and uniqueness of solutions for the governing partial differential equation with a nonlinear boundary condition. Next, an existence proof of an optimal solution was given. Then, we derived a necessary condition of optimality. Finally, a finite element algorithm was developed and optimal error estimates derived. See [4].

List of Publications and technical reports published

[1] *Temperature Distribution in Extrusion Processes.*
with L. S. Hou (York University)
Proceedings of the First World Conference in Applied Computational Fluid Mechanics, May 1992

[2] *Finite Element Approximation of Optimal Control Problems for the Von Karman Equations (Part I)*
with L. S. Hou
ICASE Technical Report No. 94-63
Institute for Computer Application in Science and Engineering
NASA Langley Research Center
July 1994

[3] *Finite Element Approximation of Optimal Control Problems for the Von Karman Equations (Part II)*
with L. S. Hou (York University)
to appear in *Numerical Methods for Partial Differential Equations*

[4] *Analysis and Finite Element Approximation of an Optimal Control Problem in Electrochemistry with Current Density Controls*
with L. Steven Hou
to Appear in *Numerische Mathematik*